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The response of an electron in a biased double well to a strong laser: delocalization and low frequency generation

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Abstract

We show that a strong laser acting on an electron in a quantum double well which is under the influence of a strong static field, will cause (for special values of the parameters) emission of intense low frequency radiation and can either completely delocalize or completely localize the electron.

The response of an electron in a double well (or that of a two level system representing a double well) to a strong laser has been the subject of several recent investigations[10]. The parameters most useful for describing the observations are, $e_0 = 2\mu_{12}E_0/\hbar\omega$, $2\varepsilon/\hbar\omega$ and the frequency:

$$\Omega_0 = (2\varepsilon/\hbar\omega)\, J_0(e_0) \tag{1}$$

Here 2ε is the energy gap between the lowest two levels, ω is the laser frequency, E_0 is the electric field of the laser, μ_{12} is the transition dipole between these levels and J_0 is the Bessel function of order zero[11]. The values of $\{\omega, E_0\}$ for which

$$\Omega_0 = 0 \tag{2}$$

are called[6-7] points of accidental degeneracy (AD points). According to (1) they satisfy

$$e_0 = 2\,\mu_{12}\,E_0/\hbar\omega \;=\; z_i(0), \tag{3}$$

where $z_i(0)$ denotes the i-th zero of J_0. If

$$2\varepsilon/\hbar\omega < 1, \tag{4}$$

the behavior of the system when $\{\omega, E_0\}$ is at or near an AD point is very interesting. As is well known, if at a given moment the electron is localized in one of the wells, it will oscillate between the wells as the time evolves. A laser with the parameters chosen at an AD point [i.e. satisfying Eq.(3)] can stop this oscillation and keep the electron in one of the wells[1,2]. Moreover, the system can emit (or absorb) intense even-harmonics[6,7] [i.e. the induced dipole $\mu(t)$ has intense Fourier components at the frequency $2n\omega$, where n is an integer]. The presence of this emission is remarkable because (for a symmetric well or, equivalently, for a two level system with zero static dipole moment) it is symmetry forbidden to all orders in perturbation theory in E_0. If $\{\omega, E_0\}$ is chosen near an AD point the Fourier transform of $\mu(t)$ has no even harmonics and has a large peak at the low frequency Ω_0. The existence of this peak is called[6-7] low frequency generation (LFG). This frequency is small, and can be continuously tuned to smaller values, by changing ω or E_0 to get them closer to an AD point. When the AD point is reached Ω_0 becomes zero and the LFG peak in $\mu(\Omega)$ disappears to become a large static dipole. Some of the work on this system assumed that the electron is initially localized. Such a state is very difficult to prepare. However, numerical calculations[6-7] show that the same behavior is obtained if the system starts in the ground state and is excited with a semi-infinite Gaussian pulse with a well chosen rise time and $\{\omega, E_0\}$ at or near an AD point.

These phenomena were documented by analysis of two level models[1,2, 9,10] and by numerical calculations with double wells[1,2,6,7]. They have also been observed in numerical calculations on systems that mimic an AlAs-GaAs-AlAs double quantum well[6-7]; the analysis of these results[9] indicates that, for the parameter range of interest here, the behaviour of the quantum double wells provided by an AlGaAs superlattice can be represented rather accurately by a two level model.

In this article we investigate how these phenomena are modified when the system is interacting with a static electric field of intensity E_s. The presence of this field introduces a new parameter

$$e_s = 2\mu_{12}E_s/\hbar\omega \tag{5}$$

The analog of the AD points (we call them generalized AD points (GAD)) are the points $\{E_s, \omega, E_0\}$ for which

$$e_s = n, \tag{6}$$

and

$$e_0 = z_i(n). \tag{7}$$

Here n is an integer (positive or negative) and $z_i(n)$ the i-th root of the Bessel function J_n. For parameter values *near or at* a GAD point the *dominant* contribution to the dipole induced by the joint action of the static field and the laser is:

$$\mu(\tau) = \mu_{12}\,\mu_n(\tau) \equiv (e_s + n)^2/W_n^{\,2} + \Omega_n^{\,2} / W_n^{\,2} \cos\{\tau W_n\} \tag{8}$$

Here $\tau = t\,\omega$, t is time,

$$\Omega_n = (2\varepsilon/\hbar\omega) J_n(e_0) \tag{9}$$

and

$$W_n = [\Omega_n^{\,2} + (n + e_s)^2]^{1/2} \tag{10}.$$

The arguments leading to equation (8) suggest that this is a good approximation for $\mu(\tau)$ only if $2\varepsilon/\hbar\omega < 1$. The above equations agree well with the induced dipole calculated numerically only for sufficiently large values of e_0; the agreement at lower values of e_0 becomes better and better as $2\varepsilon/\hbar\omega$ is made smaller.

Note that in the absence of the bias we have $e_s = 0$. The condition (6) can then be satisfied only if $n = 0$, which means that $W_n = \Omega_n = \Omega_0$; the condition (7) becomes equivalent to Eq. (3), as it should. When n is zero, Eq. (8) is valid for small nonvanishing values of e_s and generalizes results obtained in ref. 9.

Before we proceed we comment on the physical restrictions imposed by these conditions and the systems which can satisfy them. The condition $e_s \leq 1$ together with $\hbar\omega > 2\varepsilon$ implies that the electron interaction with the static field must be larger that the gap 2ε between the two levels. This condition cannot be satisfied by atoms or molecules but are satisfied in quantum wells because the barrier is low (the electron solid interaction is weak) and the system is large (the electron kinetic energy can be low); perhaps a carefully chosen proton tunneling system might also satisfy it. The condition $\hbar\omega > 2\varepsilon$ is also troublesome. The system can be treated as a two level system only if $\hbar\omega << \varepsilon_3 - \varepsilon_1$. These two conditions require that $\varepsilon_3 - \varepsilon_2 >> \varepsilon_2 - \varepsilon_1$, which is easily satisfied for a quantum well or a tunneling system, but not for an atom or a molecule.

The results summarized above are derived by analysing the equation of motion for the induced dipole. We show that $\mu(\tau)$ is the sum of the result shown in Eq. (8) and a term $(2\varepsilon/\hbar\omega)^2 c(\tau)$, where $c(\tau)$ is a functional of $\mu(\tau)$). We then argue that, if the parameters are sufficiently close to a GAD point and $2\varepsilon/\hbar\omega < 1$ than $(2\varepsilon/\hbar\omega)^2 o(t)$ is small and Eq. (8) is a good approximation.

What follows is a brief outline of the reasoning that lead us to Eq. (8). We use the model Hamiltonian:

$$H = \varepsilon\{|2\rangle\langle 2| - |1\rangle\langle 1|\} - \{|1\rangle\langle 2| - |2\rangle\langle 1|\}\mu_{12}[E_s+E_0\cos(\omega t)]. \quad (11)$$

Here $|1\rangle$ and $|2\rangle$ are the ground and first excited states of the electron in the symmetric double well in the absence of the laser and of the bias, and $2\varepsilon = \varepsilon_2 - \varepsilon_1$. The zero of energy is half-way between the ground and the excited state.

The induced dipole is:

$$\mu(t) = \mu_{12}\langle\psi,t|(|1\rangle\langle 2| - |2\rangle\langle 1|)|\psi,t\rangle, \quad (12)$$

where $|\psi,t\rangle$ satisfies the time dependent Schrödinger equation with the Hamiltonian (11). By using standard methods we derive for $\mu(t)$ the equation of motion:

$$d\mu(\tau)/d\tau = -(2\varepsilon/\hbar\omega)^2 \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_m(e_0)J_k(e_0)$$

$$\mathrm{Re}\left\{\int_0^\tau d\tau_1 \exp[i(m+e_s)\tau] \exp[i(k+e_s)\tau_1]\,\mu(\tau_1)\right. \quad (13)$$

Eq. (13) is used to search for points in the parameter space where the induced dipole may have unusual behavior. If we replace in the rhs $\mu(\tau)$ with μ_{12}, we can perform the integral and find that the terms corresponding to $m = k = -e_s$ lead to secular terms (i.e. they grow indefinitely with τ) which grow faster with time than the other terms. This suggests that when we do not use perturbation theory the terms corresponding to $m = k = -e_s$ may be larger than the rest[9].

To find the contribution to $\mu(\tau)$ from the terms $m = k = -e_s$ we must calculate them nonperturbatively. This is done as follows. We assume that $e_s = -n$ where n is an integer. Then we write the rhs of Eq. (13) as the term corresponding to k = n plus all the other terms which are $(2\varepsilon/\hbar\omega)^2 c(\tau)$; these include the terms where $m \neq k$ and the terms where $m = k$ but $m \neq n$. This separation allows us to write Eq. (13) as:

$$d\mu(\tau)/d\tau = -\Omega_n^2\{\int_0^\tau d\tau_1 \cos[(n+e_s)(\tau-\tau_1)]\mu(\tau_1)\} - (2\varepsilon/\hbar\omega)^2 c(\tau). \tag{14}$$

We can now use the Laplace transform method to "solve" this equation. We use the initial condition:

$$\mu(t=0) = \mu_{12}, \tag{15}$$

because in a biased system in which $\mu_{12}E_s >> \varepsilon$ the dipole moment in the ground state (in the absence of the laser) is very close to $\mu_{12} = e\langle 1|x|2\rangle$. Here e is the electron charge, x is the electron position operator and $|1\rangle$ and $|2\rangle$ are the eigenstates of the unbiased system.

Laplace transforming (14), using (15), solving for the Laplace transform of μ, and then inverse-transforming gives:

$$\mu(\tau) = \mu_{12}\,\mu_n(\tau) - (2\varepsilon/\hbar\omega)^2 \int_0^\tau d\tau_1\, \mu_n(\tau_1)\, c[\tau-\tau_1] \tag{16}$$

This equation is exact, but it does not provide a solution for our problem since $c(\tau)$ depends of μ. The result is however useful because the last term at the right hand side is negligible when $(2\varepsilon/\hbar\omega)$ is small; thus (16) leads to (8). The argument suggesting that the last term is negligible is patterned after a procedure presented in Ref. 9. But since it is too elaborate to be presented here we prefer to test this suggestion numerically.

Eq. (8) predicts that if $e_s \approx -n$ than $\mu(\Omega)$ is dominated by two modes, a zero frequency mode (ZFM) and a low frequency mode (LFM); furthermore, the equations gives formulae for the amplitudes of the modes and the low frequency W_n. To test these results we have calculated $\mu(\tau)$ numerically, found its Fourier transform $\mu(\Omega)$ and plotted $|\mu(\Omega)|$ as a function of Ω. These plots have peaks whose heights give the amplitude of the Fourier components of μ. We find that as long as $2\varepsilon/\hbar\omega << 1$ and $e_s \approx -n$, the ZFM and the LFM modes have the highest amplitudes, in qualitative agreement with Eq. (8). For a quantitative test we plot in Fig. 1 the amplitude of the ZFM (denoted $\mu(0)$), the amplitude of the LFM (denoted $\mu(W_1)$ and the lowest frequency W_1, as a function of e_0. The numerical calculations were made for $e_s = -1.00$ and $2\varepsilon/\hbar\omega = 0.129$. For these values Eq. (8) predicts that $\mu(0) = 0$, $\mu(W_1) = 1$ and $W_1 = (2\varepsilon/\hbar\omega)J_1(e_0)$. We plot these predictions along with the numerical results. The predictions are reasonably accurate if $e_0 > 1$, and excellent for $e_0 > 2$. The Eq.(9), giving the LFG emission frequency, agrees extremely well with the value obtained numerically, except when e_0 is very small; Eq. (9) predicts that $W_1 \to 0$ if $e_0 \to 0$, while the numerical results lead to a finite value.

As the intensity of the laser field is increased $\mu(0)$ becomes smaller while $\mu(W_1)$ increases. The sum of these two components is close to the maximum possible value μ_{12} of the induced dipole. This means that the contributions from the other Fourier components are substantially smaller, as predicted by the theory. As e_0 approaches 1 the oscillating part dominates the dipole. This means that practically all the charge density is set in motion by the laser. This coherent motion of the charge density leads to intense LFG. The frequency W_1 varies with e_0 (but it is always much smaller than the typical frequencies appearing in the Hamiltonian) and approaches zero as e_0 gets close to the first zero of J_1 (which is a GAD point). When $W_1 = 0$ the LFM becomes static. An experiment that monitors only the static part of $\mu(t)$ will notice that the static dipole disappears as e_0 exceeds one and reappears suddenly when $e_0 = z_1(1) = 3.84$.

Note that the analytical results disagree with the numerical ones if $e_0 < 1$; for this values of e_0 the terms neglected in Eqs. (14,16) become important. The numerical calculations show that the values of e_0 below which Eq. (8) is erroneous become smaller as $2\varepsilon/\hbar\omega$ is diminished.

The theory makes predictions also for the case when e_s is close to an integer. In Fig. 2 we compare the theory with the numerical results for $e_s = -0.95$. The agreement is rather good for $1<e_0$ and excellent for $2<e_0$. As the laser

intensity is increased the amplitude $\mu(0)$ of the ZFM is diminished but it never becomes zero (as it does when $e_s=-1$). When e_0 approaches the first zero of J_1, $\mu(0)$ goes up to a maximum. At this point the electron is completely localized.

We have also studied the case $e_s= - 1.05$. The equation (8) predicts that the results should be identical to those obtained for $e_s= - 0.95$, but we find this to be only approximately true. Differences between the calculation for $e_s= - 1.05$ and that for $e_s= - 0.95$ are small but not zero; they become negligible when $e_0 > 2.5$.

The analytical results are valid only if e_s is close or equal to an integer. Numerical tests show that they fail rather badly in the case when $e_s= - 1.5$.

In the case of an unbiased well, studied in previous work[1-10], the ability of the laser to prepare a state having the largest possible static moment (which means that the electron is located in one well) is rather startling. The double well studied here interacts with a very large static field (i.e. $\mu_{12} E_s>2\varepsilon$). Because of this the system has, in the absence of the laser, a very large static dipole and the electron is localized. In this case the surprise is that, in spite of an extremely strong bias, the laser manages to make the *static* component of the dipole very small, by forcing most of the charge density to slosh coherently back and forth between the wells.

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Figure Captions

Fig. 1. The zero frequency and the low frequency components of the induced dipole moment ($\mu(0)$ and $\mu(W_1)$) and the lowest frequency W_1, as functions of e_0. The continuous lines are the numerical results. The values of W_1, $\mu(0)$ and $\mu(\Omega_1)$ calculated usir。 Eq. 8 are shown as dots, squares and x's, respectively. 2ε = 12.9 cm^{-1}, ω = 100 cm^{-1}, and e_s = 1.0.

Fig. 2. Same as Fig. 1 except that e_s = 0.95.

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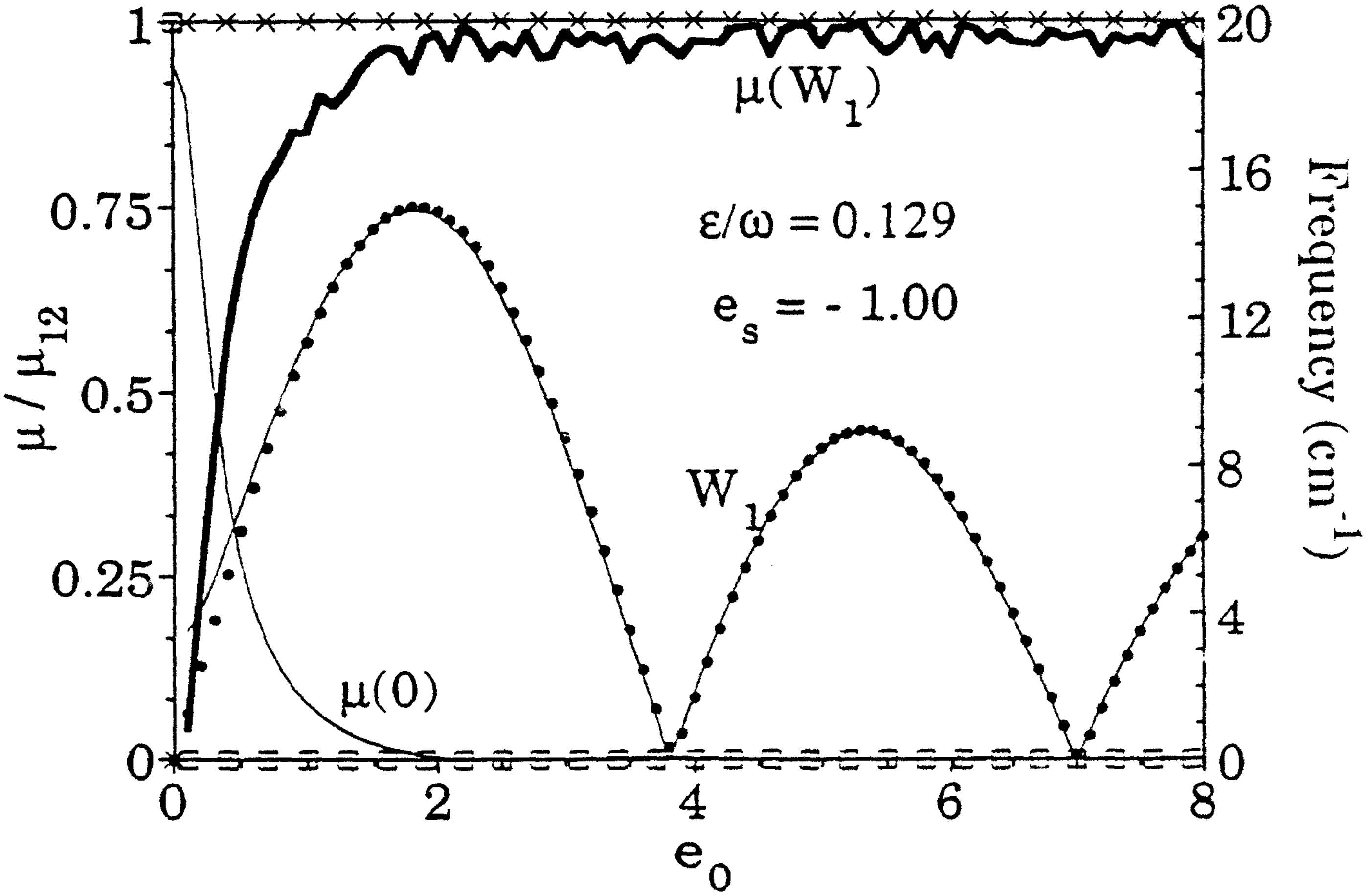
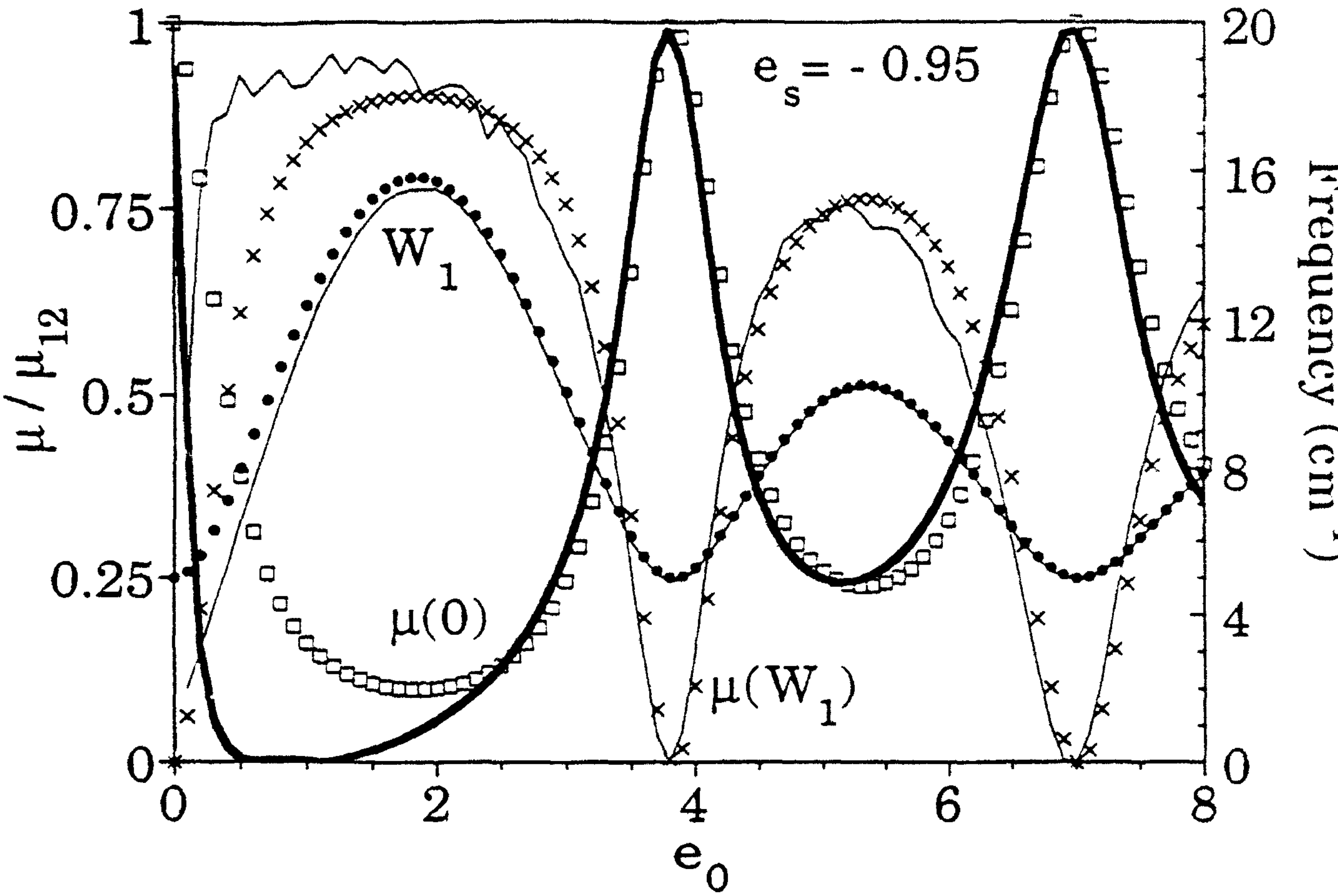
13. A more detailed description of this type of procedure can be found in ref. 9, where it has been applied to the system in the absence of the bias.



μ / μ12
Frequency (cm-1)
e0
μ(W1)
ε/ω = 0.129
es = - 1.00
W1
μ(0)
0
0.25
0.5
0.75
1
2
4
6
8
12
16
20



Dikhrovski Boufa

[illegible] Fig. 2

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